Thermal drive in centrifugal fields—mixed convection in a vertical rotating cylinder

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Abstract—Axis-symmetric secondary flows occur in a vertical rotating cylinder with a temperature difference imposed on the top and bottom ends even under zero gravity and, consequently, enhance the heat transport between two ends. The mean equivalent conductivity for mixed convection in the vertical rotating cylinder increases at first and then decreases with the rotating Reynolds number. A dimensionless parameter group $Gr/(36Re^{1.255})$ to replace Gr/Re^2 , which is commonly used in mixed convections, is suggested to measure the relative importance of buoyancy and inertial force on the mean equivalent conductivity.

1. INTRODUCTION

MANY STUDIES have been made of natural convection, which is well known as a kind of thermally-induced fluid motion, namely thermal drive in a gravitational field. In contrast, there are still several open problems on the thermal drive in miscellaneous force fields other than gravity, for instance, centrifugal, surface tension and electromagnetic fields, although they may often appear in single crystal growth in space or on Earth, the cooling of rotating machinery or the centrifuges for isotope separation and the stability of arc plasma, etc. [1].

In centrifugal fields, the original force equilibrium in fluids may be destroyed when a temperature difference is imposed and secondary flows are then produced by body forces such as centrifugal and/or Coriolis forces. As a result, the primary velocity and temperature profiles are affected by secondary flows. Mori and Nakayama [2] have given a general review of the secondary flows in rotating ducts with emphasis on its effects on the heat transfer rate of the systems. For a vertical annulus with a rotating inner cylinder, it is widely known that a critical speed of rotation exists, above which occurs a stable, secondary flow consisting of regularly spaced toroidal vortices. This flow is commonly referred to as Taylor-Couette flow, resulting from an inherent hydrodynamic instability. A comprehensive review of the general Taylor problem was provided by DiPrima and Swinney [3]. As stated by Ball et al. [4], the first efforts to study the combined problem with rotation and natural convection were usually coupled with an axial flow through the annular gap [5-8]. The global heat transfer rates were reported, corresponding to the different flow regime. A numerical study was conducted of heat transfer in a vertical rotating annulus at low Reynolds numbers by Leonardi et al. [9]. It was found that there exists a critical value of Gr/Re^2 above which rotation does not influence the rate of heat transfer.

Ball *et al.* [4] carried out an experimental study of heat transfer in a vertical annulus with a rotating inner cylinder. In rotating systems, the existence of hydrodynamic instabilities may lead to a variety of secondary flows as the parameters describing the system are varied, which usually results in markedly changed heat transfer. It was observed that the qualitative behavior of the mixed convection flows and consequent heat transfer are entirely dependent upon the value of Gr/Re^2 .

In this paper, a mixed convection in a vertical rotating cylinder with a temperature difference imposed on the top and bottom ends is investigated. The emphasis is on the physical mechanism of thermal drive and its reverse effects on the heat transfer rate under zero gravity, and under combined gravitational and centrifugal fields. Finally, dimensional analysis and numerical results show that the gravity-induced convection can be neglected if $Gr/(36Re^{1.255})$, rather than Gr/Re^2 , is below its critical value.

2. MATHEMATICAL FORMULATION AND SOLUTION PROCEDURE

Consider a vertical cylinder of radius R and height L filled with air and enclosed by ends, as shown in Fig. 1. The rotation of the cylinder together with the ends round its axis will result in a motion of air in the cylinder. The air motion under the isothermal condition behaves as a 'rigid body rotation'. An axial flow (sometimes called secondary flow) may be produced if a temperature difference is imposed between the top and bottom ends. Assume that the fluid properties are temperature independent except for the density in the body force terms (centrifugal force and Coriolis force as well as gravity). This is an extension of the Boussinesq approximation usually adopted in the numerical study of natural convection. Thus, the governing equations for a steady, laminar flow with negligible

NOMENCLATURE			
g	acceleration due to gravity	Ξ	axial coordinate.
Gr	Grashof number		
k	thermal conductivity		
$K_{\rm eq}$	local equivalent conductivity,	Greek symbols	
	$(\partial \Theta/\partial z) _{z=0,1}$	χ	thermal diffusivity
Kegm	mean equivalent conductivity,	β	thermal expansion coefficient
	$\left[2\int_{0}^{R}r(\partial\Theta/\partial z)\mathrm{d}r\right]/R^{2}$	Γ	aspect ratio
Р	pressure	θ	circumferential coordinate
Pr	Prandtl number	Θ	dimensionless temperature difference
$Q_{\rm c}$	circulating flow rate, $\pi \int_{0}^{R} V_{z} dr$	v	kinematic viscosity
r	radial coordinate	ρ	density
R	cylinder radius	σ	densiometric Froude number, Gr/Re^2
Re	rotational Reynolds number	σ^*	revised densiometric Froude number,
Т	temperature		$Gr/(36Re^{1.255})$
$T_{\rm h}$	bottom end temperature	ϕ	scalar-dependent variable (e.g. Θ ,
$T_{\rm e}$	top end temperature		V_r, V_r)
V_r, V_r	V_{θ} velocity components in the r-, z-	Ψ	dimensionless stream function
	and θ -directions	ω	rotational speed.

viscous dissipation in the rotating cylinder may be written as follows:

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\mathbf{V} \cdot \nabla V_r = \frac{\rho}{\rho_0} \frac{V_\theta^2}{r} - \frac{1}{\rho_0} \frac{\partial P}{\partial r} + \nu \left(\nabla^2 - \frac{1}{r^2} \right) V_r \quad (2)$$

$$\mathbf{V} \cdot \nabla V_{\theta} = -\frac{\rho}{\rho_0} \frac{V_r V_{\theta}}{r} + \nu \left(\nabla^2 - \frac{1}{r^2}\right) V_{\theta} \qquad (3)$$

$$\mathbf{V} \cdot \nabla V_z = \left(1 - \frac{\rho}{\rho_0}\right) g - \frac{1}{\rho_0} \frac{\partial P}{\partial z} + v \nabla^2 V_z \qquad (4)$$

$$\nabla \cdot (\mathbf{V}T) = \alpha \nabla^2 T. \tag{5}$$

Using R, $R \cdot \omega$, $\rho_0 R^2 \omega^2$ and $(T_h - T_c)$ as scale factors for length, velocity, pressure and temperature, respectively, the dimensionless quantities are defined as



FIG. 1. The geometry and the coordinate system.

$$Fr = R\omega^2/g, \quad Gr = \frac{g\beta(T_{\rm h} - T_{\rm c})l^3}{v^2}$$

where R is the radius of the cylinder, ω is the angular speed of rotation of the cylinder, T_h and T_c are the bottom and top end temperatures, respectively, ρ_0 represents the air density at ambient condition, Re is the rotating Reynolds number, Gr the Grashof number and Fr the Froude number. Using the original symbols, except for the above-cited non-dimensional parameters, equations (1)-(5) become

$$\vec{\mathbf{v}} \cdot \mathbf{V} = 0 \tag{6}$$

$$\mathbf{V} \cdot \nabla V_r = \left(1 - \frac{Gr \cdot Fr}{Re^2} \Theta\right) \frac{V_\theta^2}{r} - \frac{\partial P}{\partial r} + \frac{1}{Re} \left(\nabla^2 - \frac{1}{r^2}\right) V_r \quad (7)$$

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$$\mathbf{V} \cdot \nabla V_{\theta} = -\left(1 - \frac{Gr \cdot Fr}{Re^2} \Theta\right) \frac{V_r V_{\theta}}{r} + \frac{1}{Re} \left(\nabla^2 - \frac{1}{r^2}\right) V_{\theta} \quad (8)$$

$$\mathbf{V} \cdot \nabla V_z = \frac{Gr}{Re^2} \Theta - \frac{\partial P}{\partial z} + \frac{1}{Re} \nabla^2 V_z \tag{9}$$

$$\nabla \cdot (\mathbf{V}\Theta) = \frac{1}{Re Pr} \nabla^2 \Theta. \tag{10}$$

The boundary conditions for the problem can be written using no-slip conditions for velocity, constant temperature on the top and bottom ends and an insulated condition on the cylinder wall. That is



FIG. 2. Streamlines and isotherms for various rotational Reynolds numbers under zero gravity.

at r = 1

$$V_r = 0 \quad V_z = 0 \quad \frac{\partial \Theta}{\partial r} = 0$$

$$V_{\theta} = 1 \quad \text{(for rotating cylinder)}$$

$$V_{\theta} = 0 \quad \text{(for stationary cylinder)}$$
at $z = 0$

$$V_r = 0 \quad V_z = 0 \quad \Theta = 1$$

$$V_{\theta} = r \quad \text{(for rotating cylinder)}$$

$$V_{\theta} = 0 \quad \text{(for stationary cylinder)}$$
at $z = 1$

$$V_r = 0 \quad V_z = 0 \quad \Theta = 0$$

$$V_{\theta} = r \quad \text{(for rotating cylinder)}$$

 $V_{\theta} = 0$ (for stationary cylinder).

The governing equations and the associated boundary conditions are discretized by a control volumebased finite difference method. The finite difference procedure adopts a power-law scheme for the convection-diffusion formulation in the present study. An ADI iterative solution procedure is employed here to obtain the steady-state solution of the problem considered.

The convergence criteria used for all field variables ϕ are as follows:

$$\frac{|\phi_{i,j}^{n+1} - \phi_{i,j}^{n}|_{\max}}{|\phi_{i,i}|_{\max}} < 10^{-4}$$

where n is the index representing iteration number. The extent of agreement between the energy input and output is also used to check the validity of the numerical scheme. The energy balance is maintained within 1%.

Uniform grids that discretize the r- and z-directions into 50 by 50 nodal points are used for the present calculations. The adequacy of the grid size selected was established on the basis of tests for numerical accuracy on the refined grids until relatively small changes in the results were obtained.

3. RESULTS AND DISCUSSION

The flow and temperature fields are calculated for air in both stationary and rotating vertical cylinders. The rotating Reynolds number of the cylinder ranges from zero to 6000 at fixed aspect ratio L/R = 1 and Grashof number varied from zero to 4.86×10^5 . Most of the numerical results are first presented in the form of streamlines and isotherm contour plots. The local and mean equivalent conductivities along the top and bottom ends are then given for the discussion on the interaction of fluid flow and heat transfer.

3.1. Thermal drive in a vertical rotating cylinder under zero gravity

As mentioned above, axis-symmetric secondary flows in the plane containing the cylinder axis appear in the vertical rotating cylinder, as shown in Fig. 2(a), if the bottom end temperature T_h is higher than the top end temperature T_c . It is worth noting that the cause of such secondary flows is mainly the centrifugal force rather than buoyancy, even in the combined centrifugal and gravitational fields. The following fact is good evidence for this argument : the same secondary flows can still be produced except for the change in their clockwise direction as the top end temperature is higher (Fig. 2(b)). The physical mechanism for fluid motion is that the imposed temperature difference between the ends causes the axial variation of centrifugal force and consequently an axial pressure gradient which can by no means be balanced by the centrifugal force itself. As a result, an axial motion of fluid must be produced.

The influence of secondary flows on the temperature field of air in the rotating cylinder is shown in Figs. 2(b) and (c). As the rotating Reynolds number Re increases, the position of the center of circulations



FIG. 3. Effect of circulating flow rate on the mean equivalent conductivity under zero gravity.



FIG. 4. Local equivalent conductivity along the ends for various rotational Reynolds numbers under zero gravity.



STREAMLINES FOR Re=0 Gr=278000 ISOTHERMS FOR Re=0 Gr=278000



FIG. 5. Streamlines and isotherms in a stationary cylinder for various Grashof numbers.

moves outwards and the streamlines become more dense in the near-wall region than in the near-axis region. The occurrence of the circulations not only intensifies the heat transport between the top and bottom ends, but also alters the form of temperature distribution—it makes the isotherms curved. This becomes more marked in the near-wall region at large Re. The intensification of heat transfer can be represented by the increase of the mean equivalent conductivity, K_{eqm} , i.e.

$$K_{\rm eqm} = \frac{2\int_0^R r \frac{\partial \Theta}{\partial z} \,\mathrm{d}r}{R^2}.$$
 (11)

The intensity of the circulations may be described by the circulating flow rate, Q_c , or

$$Q_{\rm c} = \pi \int_0^R |V_z| \, \mathrm{d}r. \tag{12}$$

It can be seen in Fig. 3 that the rising rotating Reynolds number leads to an increase of the circulating flow rate and consequently the mean equivalent conductivity. The local equivalent conductivity along the top and bottom ends shown in Fig. 4 corresponds to the local intensity of circulation, which reaches its maximum at r = R.

3.2. Buoyancy-induced flow in a stationary cylinder

Consider a stationary cylinder with the temperature of the bottom end higher than that of the top end, with no heat transfer through the cylinder wall. The fluid in the enclosed cylinder starts to move and circulating flows form as the Grashof number increases above its critical value. Its meridional flow pattern is illustrated in Fig. 5(a). With the Grashof number increasing, the combined action of inertial and viscous forces initiates a reverse flow at the top corner, shown in Fig. 5(b). When Gr is large enough, there exist bicellular flows nearly equal in size and opposite in direction and, as a result, symmetric isotherms (Fig. 5(c)).

Both the local and mean equivalent conductivities are largely dependent on the flow pattern. Figure 6 shows that the local equivalent conductivities for two ends are quite different at relatively small Gr, while they are almost identical at large Gr, which corresponds to the case of symmetric circulations. The dependence of the mean equivalent conductivity for



FIG. 6. Local equivalent conductivity along the ends in a stationary cylinder for various Grashof numbers.



FIG. 7. Variation of mean equivalent conductivity in a stationary cylinder with Grashof number.

ends on Gr is given in Fig. 7. As anticipated, K_{eqm} initially increases with Gr because of the increase of circulating flow rate. However, as long as the reverse flows occur, more viscous dissipation makes the flow velocity lower and hence leads to the decrease of heat transfer. These are the results of variation of flow pattern. The peak of K_{eqm} in Fig. 7 represents the case just before the onset of the reverse flow. The formation of bicellular flows with equal size and opposite direction corresponds to the situation of minimum equivalent conductivity, as shown in Fig. 7. Afterwards, the equivalent conductivity rises monotonically with the Grashof number due to the increase of the circulating flow rate without variation in the flow pattern.

3.3. Mixed convection

The flow pattern in a stationary cylinder under gravity, shown in Fig. 5(c), will gradually change when the cylinder begins to rotate. The centrifugal force assists the clockwise circulation at the top but retards the anti-clockwise circulation in the bottom, as shown in Fig. 8(a). With the further increase of rotating speed, the flow pattern becomes four horizontal ranked circulations (Fig. 8(b)). When the rotating Reynolds number is large enough, only one circulation is left and its center moves toward the cylinder wall (Fig. 8(c)). Figure 9 shows the local equivalent conductivities along the ends under different rotating Reynolds numbers. The flow pattern of four horizontal ranked circulations results in a wavelike distribution of local equivalent conductivity. In the case of high rotating Reynolds number there exists single circulation only and the circulating flow rate becomes much smaller in the central region than that in the near-wall region, which leads to the lower and higher local equivalent conductivity in the central region and in the near-wall region, respectively.

The dependence of the mean equivalent conductivity, K_{eqm} , of the cylinder on the rotating Reynolds number with constant Grashof number is given in Fig. 10. K_{eqm} increases with *Re* starting from zero, as in

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STREAMLINES FOR Re=150 Gr=278000



ISOTHERMS FOR Re=150 Gr=278000

(a)



STREAMLINES FOR Re=1000 Gr=278000



ISOTHERMS FOR Re=1000 Gr=278000

(b)



STREAMLINES FOR Re=2500,Gr=278000



ISOTHERMS FOR Re=2500 Gr=278000

(c)

FIG. 8. Streamlines and isotherms in a vertical cylinder for various rotational Reynolds numbers; $Gr = 2.78 \times 10^5$.



FIG. 9. Local equivalent conductivity along the ends for various rotational Reynolds numbers; $Gr = 2.78 \times 10^5$.

mixed convections of other types. This is due to the fact that the horizontal ranked circulations caused by cylinder rotation are able to transport heat between the ends more directly and efficiently than the vertical ranked circulations caused by buoyancy. What differs from the other types of mixed convections is that K_{eqm} decreases with increasing *Re* beyond a certain value. This is because the natural convection is suppressed by the rotations of high speed, which can be confirmed by the rapid drop of the circulating flow rate with the increase of *Re* (Fig. 11). Furthermore, K_{eqm} will approach its value under zero gravity; that is, the contribution of natural convection disappears if *Re* is sufficiently high.

The dimensionless parameter $\sigma = Gr/Re^2$ is usually used to describe the relative importance of the buoyancy to inertial force in mixed convections. Our results suggest, however, that the relative contribution of the buoyancy and centrifugal force to the mean equivalent conductivity not only depends on σ , but also on Reitself (Fig. 12). It can also be found from equations (6)–(10) that



FIG. 10. Variation of mean equivalent conductivity with rotational Reynolds number for different Grashof numbers.



FIG. 11. Effect of circulating flow rate on the mean equivalent conductivity for $Gr = 2.78 \times 10^5$.

$$K_{\text{eqm}} = K_{\text{eqm}} \left(\frac{Gr}{Re^2}, Fr, Re, Pr \right).$$
 (13)

It turns out that K_{eqm} depends mainly on Gr/Re^2 and Re since Fr is a function of Re for a given gas with a fixed geometry and Pr is a constant for a given gas. It has been found through a sophisticated rearrangement of numerical results that the dimensionless parameter $Gr/(36Re^{1.255})$ can measure the relative importance of natural convection much better than σ ($=Gr/Re^2$), as shown in Fig. 13. For values of $Gr/(36Re^{1.255}) \leq 1$ the rotation effects dominate the flow and buoyant effects can be neglected, and the peak values of K_{eqm} for various Gr are located universally at $Gr/(36Re^{1.255}) = 3$.

4. CONCLUDING REMARKS

(1) Axis-symmetric secondary flow will occur in a vertical rotating cylinder under zero gravity, if a temperature difference (no matter if it is positive or negative) is imposed at the top and bottom ends which enclose the cylinder with an adiabatic side wall. The



FIG. 12. Variation of mean equivalent conductivity with $1/\sigma$ for different Grashof numbers.



FIG. 13. Variation of mean equivalent conductivity with $1/\sigma^*$ for different Grashof numbers.

thermal drive of this kind can enhance the heat transport in the vertical rotating cylinder. With the increase of the rotating Reynolds number, the circulating flow rate rises, which results in the increase of the mean equivalent conductivity and the center of circulations moves toward the side wall, which leads to the local equivalent conductivity at the near-wall region being much higher than that in the central region.

(2) With the rise of the Grashof number, buoyancyinduced flow in a stationary cylinder with an adiabatic side wall and two ends heated from below experiences a change in flow pattern from single circulation to two vertical ranked circulations, which weakens the heat transport between the two ends. As a result, the mean equivalent conductivity is a non-monotonic function of the Grashof number.

(3) The mean equivalent conductivity for the mixed convection in the vertical rotating cylinder initially increases with the rotational Reynolds number due to the variation of flow pattern, then decreases with the rotational Reynolds number due to the rapid drop of the circulating flow rate, which implies the suppression of natural convection by the rotational effects.

(4) A dimensionless parameter $Gr/(36Re^{1.255})$ is suggested to measure the relative importance of buoyancy and inertial force to the mean equivalent conductivity for the mixed convection in the vertical rotating cylinder. For values of $Gr/(36Re^{1.255}) \le 1$ natural convection can be ignored and for $Gr/(36Re^{1.255}) = 3$ the mean equivalent conductivities for the mixed convection reach their maximum.

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CHAMPS D'ENTRAINEMENT CENTRIFUGE THERMIQUE—CONVECTION MIXTE DANS UN CYLINDRE VERTICAL TOURNANT

Résumé—Des écoulements secondaires axisymétriques se produisent dans un cylindre vertical tournant à différence de température imposée sur les extrémités supérieure et inférieure même en l'absence de gravité et, en conséquence, cela accroît le transfert thermique entre les deux extrémités. La conductivité équivalente moyenne pour la convection mixte dans le cylindre vertical tournant augmente d'abord puis diminue ensuite avec le nombre de Reynolds croissant. Le groupe sans dimension $Gr/(36Re^{1.255})$ est proposé, à la place de Gr/Re^2 qui est couramment utilisé en convection mixte, pour mesurer l'importance relative du flottement et de l'inertie sur la conductivité équivalente moyenne.

THERMISCHER ANTRIEB IN ZENTRIFUGALEN FELDERN-MISCHKONVEKTION IN EINEM SENKRECHTEN ROTIERENDEN ZYLINDER

Zusammenfassung—In einem senkrechten rotierenden Zylinder mit einer aufgeprägten Temperaturdifferenz zwischen oberer und unterer Stirnseite entstehen, sogar unter Schwerelosigkeit, achsensymmetrische Sekundärströmungen, die den Wärmetransport zwischen den beiden Enden verbessern. Die äquivalente mittlere Wärmeleitfähigkeit für die Mischkonvektion in dem senkrechten rotierenden Zylinder nimmt zunächst mit steigender Rotations-Reynolds-Zahl zu, um dann später wieder abzunehmen. Es wird ein dimensionsloser Parameter $Gr/(36Re^{1.255})$ als Ersatz für den üblicherweise bei Mischkonvektion verwendeten Parameter Gr/Re^2 vorgeschlagen, um die relative Bedeutung der Auftriebsund Trägheitskräfte für die äquivalente mittlere Wärmeleitfähigkeit geeignet zu berücksichtigen.

ТЕПЛОПЕРЕНОС В ПОЛЯХ ЦЕНТРОБЕЖНЫХ СИЛ. СМЕШАННАЯ КОНВЕКЦИЯ В ВЕРТИКАЛЬНОМ ВРАЩАЮЩЕМСЯ ЦИЛИНДРЕ

Аннотация — В вертикальном вращающемся цилиндре при наложении разности температур на верхний и нижний торцы даже в условиях невесомости возникают осесимметричные вторичные течения, что увеличивает теплоперенос между торцами. Средняя эквивалентная теплопроводность при смешанной конвекции в вертикальном вращающемся пилиндре сначала уменьшается, а затем увеличивается с ростом вращательного числа Рейнольдса. Для определения влияния подъемной и инерционной сил на среднюю эквивалентную теплопроводность предлагается критерий *Gr*(36*Re*^{1,255}) вместо *Gr*/*Re*², обычно используемого при смешанной конвекции.